ARTICLE NO. 80-4056

J80-121

Determining Size Distribution of Liquid Nitrogen Particles Flowing in an Airstream by Scattered Light Detection

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M. E. Essawy*
Ecole Nationale Supérieure de l'Aéronautique et de l'Espace, Toulouse, France
and
A. G. Delfour†
Office National d'Etudes et de Recherches Aérospatiales, CERT, Toulouse, France

A modified method is applied to the determination of size distributions and total volumetric concentration of liquid particles flowing in an airstream, from Mie scattered-light measurements. The great disadvantage of the method of Twomey is the use of a priori assumptions about the shape of radius distributions. It is chosen randomly, which is why it yields large computing times. The present method applies Laguerre polynomials to the determination of radius distributions. The inversion technique of Twomey is then developed using Gauss-Laguerre quadrature for determination of size distributions and total volumetric concentration. The present method exhibits an order-of-magnitude reduction in computing time over that of Twomey, with comparable accuracy. This method can be applied in cryogenic wind tunnels and to the determination of size distributions in aerosols.

Introduction

SIZE distributions of aerosols or of liquid particles flowing in air can be determined from optical scattering measurements. A laser beam passes through the aerosol perpendicular to its axis and the scattered light is detected and registered by an oscilloscope or on magnetic tape. Measured phenomena represent a parameter in an "improperly posed" or "ill-conditioned" integral equation where the kernel is determined by the Mie theory.

There are several approaches to attack the problem; among them one can find the method of Phillips,[1] the method of Twomey,[1,2] and the inversion technique of Backus-Gilbert.[1,3] The great disadvantage of Twomey's method is the random choice of the radius distribution of particles. Sometimes one has to try more than 100 different distributions to obtain an acceptable one. Consequently, the computing time is large.

This paper presents an approach that modifies the method of Twomey. The radius distributions required for the determination of aerosol size distributions are calculated as the roots of Laguerre polynomials[4] $L_n(x)$. Gauss-Laguerre quadrature[4] is then applied to the integral equation. The inversion technique of Twomey[1] is then used for determination of size distributions of aerosols. The volumetric concentraion of particles in an aerosol is also determined by using a Gauss-Laguerre quadrature.

Results obtained by the present method are compared with those of the method of Twomey. The accuracy of this method is comparable; moreover, it is faster than the other. The IRIS 80 digital computer calculates four cases each with 40 different values of Lagrange multiplier within 30 s. Size distribution functions obtained by the present method are continuous functions, but those determined by Twomey are step functions. This method is applied to the determination of size distributions and total volumetric concentration of liquid nitrogen particles carried in the airflow of a cryogenic wind tunnel.

Received June 25, 1979; revision received Nov. 28, 1979.

Index categories: Airbreathing Propulsion; Aerodynamics; Cryogenics

*Research Engineer, Group of Aerodynamics. Member AIAA.
†Research Engineer, Group of Optics.

Theoretical Concepts

The intensity $I(\theta)$ of light scattered by a spherical particle of radius r in a given direction θ is proportional to the differential scattering cross section $\sigma(r,\theta)$. Let $f(r)$ be the distribution function of particles. The total number of particles per unit volume is then equal to $\int_0^\infty f(r)\mathrm{d}r$. The scattering cross section per unit volume in the θ direction is then expressed as

$$\beta(\theta) = \int_0^\infty \sigma(r,\theta)\cdot f(r)\,\mathrm{d}r \tag{1}$$

References 1, 5, and 6 present these concepts in a more detailed form but they are reviewed briefly in this section.

Equation (1) is an integral equation of the Fredholm type but is "improperly posed" or "ill-conditioned."[7] The term $\beta(\theta)$ is determined experimentally from the measurements of $I(\theta)$. The kernel of Eq. (1), i.e., $\sigma(r,\theta)$, is calculated by using the Mie theory for given wavelength of the incident laser beam and given index of refraction of the tested particles. The unknown distribution function $f(r)$ is to be determined.

Reference 1 presents several solutions based on discretization of the problem. The distribution function $f(r)$ is considered constant in the interval $(r_i - r_{i+1})$ and has the value of $f(r_i)$. Equation (1) then has the form

$$\beta(\theta_j) = \sum_{i=0}^{n} f(r_i) \int_{r_i}^{r_i+1} \sigma(r,\theta_j)\,\mathrm{d}r$$

A matrix equation of the form $B=AF$ is to be solved, where A is the matrix form of

$$a_{ij} = \int_{r_i}^{r_i+1} \sigma(r,\theta_j)\,\mathrm{d}r$$

and B and F are vectors of $\beta(\theta_j)$ and $f(r_i)$, respectively. Unfortunately, the matrix A can be singular ($\det A \sim 0$) and its direct inversion gives mathematically unstable solutions.[7] The method of Twomey presented in Ref. 1 has the following form:

$$F = (A^TA + \gamma C^TC)^{-1}A^TB \tag{2}$$

where C is a constant matrix required to determine the second derivative and γ is the Lagrange multiplier ($\gamma = 0$ corresponds to the least-square method).

For determination of the matrix F one has to propose randomly a radius distribution in advance. All calculations are to be carried out based on the chosen distribution. F is then determined from Eq. (2). The trial is repeated several times until a solution that satisfies the following conditions is obtained:

1) To be physically acceptable all values of F are positive. This is a great advantage of Twomey's method as one may have to try more than 100 different proposed distributions to find an acceptable solution.

2) The maximum difference between the measured and recalculated values of $\beta(\theta_i)$ is within 10%, which corresponds to a difference in log $\beta(\theta_i)$ of less than 0.050. This error is due to the speckle of the laser beam. "Speckle" is the phenomenon of random interferences due to the coherence of the laser light.

The present method determines the size distribution of particles in the following way: After considering that the radius of the tested particles $r = K \cdot x$ and then multiplying and dividing the kernel of the integral equation (1) by e^x, one can rewrite Eq. (1) as:

$$\beta(\theta) = \int_0^\infty e^{-x} Y(x) \, \mathrm{d}x \tag{2a}$$

where

$$Y(x) = K \cdot \sigma(r,\theta) \cdot F(r)$$

$$F(r) = e^x f(x), \quad K = \text{scale factor}$$

The Gauss-Laguerre quadrature of the form[4]

$$\int_0^\infty e^{-x} f(x) \, \mathrm{d}x = \sum_{j=1}^{n} H_j f(a_j) \tag{3}$$

is applied to Eq. (2a), which results in the equation

$$\beta(\theta_i) = \sum_{j=1}^{n} A_{ij} G_j \tag{4}$$

where

$$A_{ij} = K \cdot H_j \cdot \sigma(r_j, \theta_i), \quad G_j = f(r) \cdot \exp(a_j)$$

The weight coefficients H_j are determined by[4]:

$$H_j = (n!)^2 / a_j [L_n^{\prime}(a_j)]^2 \tag{5}$$

where $L_n(a_j)$ are Laguerre polynomials of order n and a_j are their roots. The order n of these polynomials is less than or equal to the number of the optical mirrors used by the measuring instrument described in the next section, depending on the number of angles θ_i of the scattered laser beam.

Reference 4 presents the numerical values of the roots of the Laguerre polynomials $L_n(x)$ and the corresponding values of the weight coefficients H_j at different values of n.

Applying the inverse technique of Twomey[1] to Eq. (4), the following matrix equation is obtained:

$$G = (A^T A + \gamma C^T C)^{-1} A^T B \tag{6}$$

Solving the matrix equation (6), the size distribution function can be determined by:

$$f(r) = G / \exp(a_j) \tag{7}$$

Considering that the tested particles are spherical, the volumetric distribution function $g(r)$ per unit volume has the form

$$g(r) = 4/3 \cdot \pi \cdot r^3 \cdot f(r) \tag{8}$$

The total volumetric concentration is defined as

$$C_T = \int_0^\infty g(r) \, \mathrm{d}r \tag{9}$$

In order to determine C_T the Gauss-Laguerre quadrature [Eq. (3)] is applied to Eq. (9)

$$C_T = \sum_{j=1}^{n} H_j W_j \tag{10}$$

where

$$W_j = K \cdot g(r) \cdot \exp(a_j)$$

The order n of the polynomial is determined according to the number of optical mirrors used by the measuring instrument. Consequently, the roots a_j and the weight coefficients H_j are found.[4] Equations (7), (8), and (10) determine the two size distribution functions and the total volumetric concentration, respectively.

As long as Eq. (1) is "improperly posed" the solution is not unique. One has to choose only the solution that gives the lowest error in the recalculated values of $\beta(\theta)$. This error must be less than 10% of $\beta(\theta)$, which corresponds to a difference in log$\beta(\theta)$ of less than 0.050.

This error due to the "speckle" of the laser beam is found to be greater than the error of the Gauss-Laguerre quadrature as the latter is expressed by the following[4]:

$$E = [(n!)^2 / (2n)!] f^{2n}(n) \tag{11}$$

It emerges that the determined size distribution that gives a maximum error in the recalculated $\beta(\theta)$ inferior to 10% is accepted whatever the error of the quadrature. It is found that this error of $\beta(\theta)$ results in an error of $f(r)$ which is equal to only 1.5%.

Experimental Concepts

References 5, 6, and 8 present a detailed description of the device used for measuring the scattering pattern of light. This is reviewed briefly in this section. A laser beam passes through the tested aerosol perpendicular to its flow axis. It coincides with the major axis of an ellipse where a dozen optical mirrors are fixed tangentially to the ellipse. The axis of the aerosol

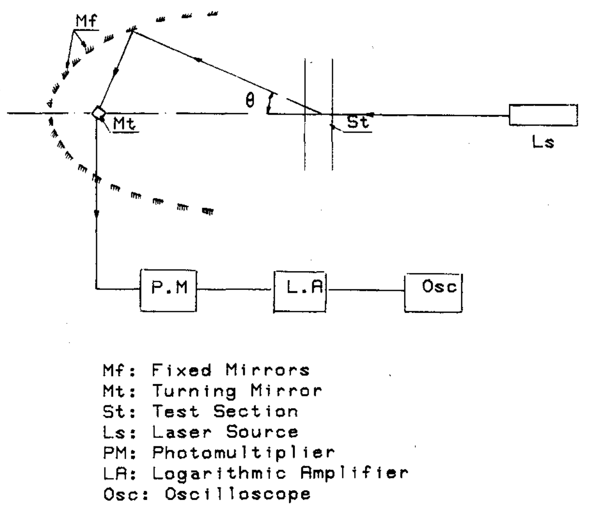

Fig. 1 Schematic of measuring device.

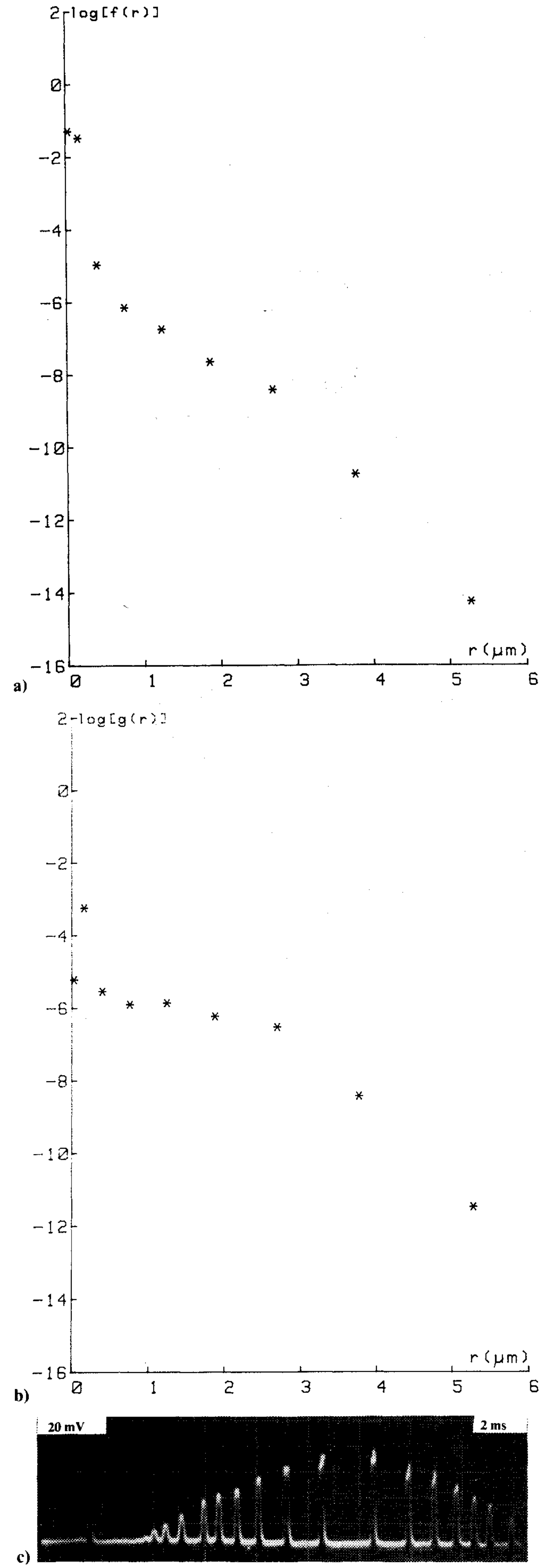

Fig. 2a Distribution function (present method); b) Volumetric distribution function; c) Scattering pattern diagram.

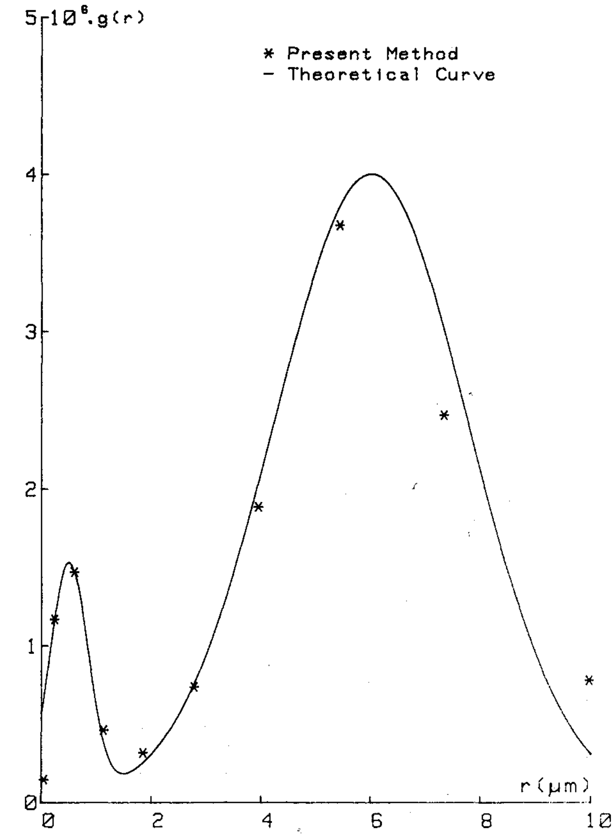

Fig. 3 Volumetric distribution function.

passes through the first focus of the ellipse and a turning mirror is located at the second focus. Light scattered by the tested particles is reflected by fixed optical mirrors towards the turning mirror. According to the geometrical characteristics of the ellipse, the distance travelled by each scattered laser beam is the same.

The turning mirror then directs all received light toward a photomultiplier, where the light is detected and then amplified by a logarithmic amplifier. The amplified signal is registered on an oscilloscope or on a magnetic tape. A simplified scheme is presented in Fig. 1.

Application of Present Method

This method has been applied to the determination of size distributions and volumetric concentration of liquid nitrogen particles carried in the airstream of a cryogenic wind tunnel. Several size distributions and volumetric concentrations can be determined during one run of the prepared computing program. This section presents some examples determined by this method.

Example 1

The scattering pattern of light, Fig. 2c, is measured experimentally. Size distributions and volumetric concentration $f(r)$, $g(r)$, and C_T, respectively, are determined by this method. To be sure that obtained distributions are acceptable solutions, the scattering pattern that corresponds to obtained solutions is recalculated. The maximum difference between measured and recalculated values of $\log\beta(\theta)$ is found to be equal to 0.034, i.e., $(\Delta\log\beta(\theta))_{max} < 0.050$. (Figs. 2a and 2b).

Example 2

The scattering pattern of light that corresponds to a given volumetric distribution function $g(r)$ is calculated theoretically. A random error ϵ is introduced to each theoretical value of $\log\beta(\theta)$, i.e., $-0.025 < \epsilon < 0.025$, and $g(r)$ is recalculated by the present method. There is good agreement between the original and recalculated distributions. (Fig. 3).

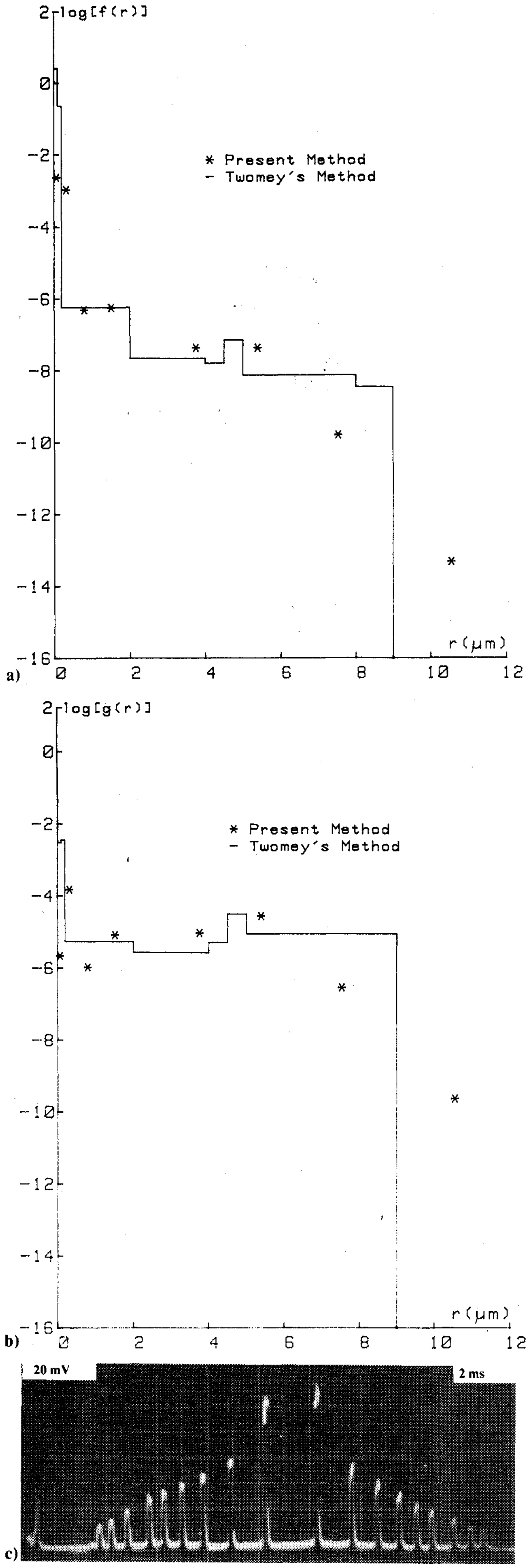

Fig. 4a) Distribution function; b) Volumetric distribution function; c) Scattering pattern diagram.

Table 1 Comparison of results

	Method	
	Present	Twomey
Volumetric concentration	6.4918×10^{-4}	6.9441×10^{-4}
$[\Delta \log \beta(\theta)]_{max}$	0.044	0.024
Calculation time	3 s	24 s

Comparison

A comparison between results obtained by this method and results determined by the method of Twomey for the same measured scattering pattern (Fig. 4 and Table 1) is presented herein. There is good agreement between both methods, but the results by Twomey's method are obtained after trying 36 different distributions of radius, each with 5 different values of Lagrange multipler (with good luck). It required 24 s of computing time by the IRIS 80 digital computer. On the other hand, the computing time required by the present method is only 3 s. Consequently, the present method is actually faster than the other with a comparable accuracy.

Conclusions

This paper presents a method for the determination of size distributions and volumetric concentrations of particles in a two-phase flow, from detecting and analyzing the light scattered by tested particles. This method has the following principal advantages:

1) No random choice of radius distribution is required. This distribution is to be calculated in advance.

2) Size distributions are obtained in the form of points and not as step functions.

3) Estimating in advance the global zone of the radius of particles, the determination of size distributions and volumetric concentration is very rapid.

4) The program of computation is able to calculate several cases of distributions and volumetric concentration during one run. Moreover, the computing time is short.

5) This method can be applied in cryogenic wind tunnels and in the determination of size distributions and volumetric concentration in aerosols generally.

Acknowledgments

The authors acknowledge the constant guidance of J. Luneau, chief of the Aerothermodynamic Department at the Ecole Nationale Supérieure de l'Aéronautique et de l'Espace (ENSAE), Toulouse, France. They are indebted to the useful support and discussions of M. Laug, the chief of the Department of Optics, ONERA-CERT, Toulouse, France.

References

[1] Frut, G., "Recherche de la Fonction de Distribution," Office National d'Etudes et de Recherches Aérospatiales, CERT, Toulouse, France, Rept. 1/6010, Feb. 1974.

[2] Twomey, S., "On the Deduction of the Vertical Distribution of Ozone by Ultraviolet Spectral Measurements from a Satellite," *Journal of Geophysical Research*, Vol. 66, July 1961, pp. 2153-2162.

[3] Westwater, E. and Cohen, A., "Applications of Backus-Gilbert Inversion Technique to Determination of Aerosol Size Distribution from Optical Scattering Measurements," *Applied Optics*, Vol. 12, June 1973, pp. 1340-1348.

[4] Kopal, Z., *Numerical Analysis*, 2nd ed., Chapman and Hall, London, 1961, pp. 347-373.

[5] Laug, M., Luneau, J., and Frut, G., "Mesure de Granulométrie dans les Ecoulements Diphasiques," 11th AAAF Colloquium, Bordeaux, France, Nov. 1974.

[6] Laug, M., Frut, G., and Aubry, Y., "Etude Granulométrique d'Aérosols Etalons par Diffusion de Lumière Laser," Office National d'Etudes et de Recherches Aérospatiales, CERT, Toulouse, France, Rept. 3/6010, Dec. 1974.

[7] Tikhonov, A. and Arsenin, V., *Méthodes de Résolution de Problèmes Mal-Posés*, MIR, Moscow, 1976 (French translation.)

[8] Laug, M., Aubry, Y., and Fima, H., "Etude Granulométrique d'Aerosols Etalons par Diffusion de Lumière Laser—Méthode d'Etalonnage et Programme d'Essais," Office National d'Etudes et de Recherches Aérospatiales, CERT, Toulouse, France, Rept. 2/6010, May 1974.